General description

There are many possible PhD projects in Arithmetic Geometry, supervised by D. Macías (sites.google.com/site/danielmaciascastillo), or in Algebraic Topology, supervised by F. Cantero (matematicas.uam.es/~federico.cantero/).

Arithmetic Geometry studies both number fields and function fields of curves over finite fields, and rational points on varieties defined over such fields, or even on Galois representations. This study relates them to special L-values. An important example is the Birch and Swinnerton-Dyer Conjecture for elliptic curves or abelian varieties. A possible PhD problem is to investigate this conjecture through the ideas of Iwasawa theory. Recently there has also been much interest in Drinfeld modules over functions fields, a source of Galois representations and L-values in positive characteristic. There are also many possible PhD problems in this new and exciting direction.

Algebraic Topology is an area of mathematics that investigates spaces using algebra. Classifying the many ways a string can sit in the Euclidean space is the subject of knot theory, a very active area of research. One of the most promising tools to study them is Khovanov homology, which was used recently by Lisa Piccirilo to prove that the Conway knot is not slice. This invariant was connected to the field of stable homotopy theory ten years ago by Lipshitz and Sarkar. The research of the student may also lie in between these two areas.

Selected bibliography

- D. Burns, D. Macías Castillo, *On refined conjectures of Birch and Swinnerton-Dyer type for Hasse-Weil-Artin L-series*, to appear in Memoirs of the American Mathematical Society.

- D. Burns, D. Macías Castillo, C. Wuthrich, *On Mordell-Weil groups and congruences between derivatives of twisted Hasse-Weil L-functions*, J. Reine Angew. Math.734 (2018) 187-228.

- F. Cantero, *Higher Steenrod squares for Khovanov homology*, Adv. Math. 369(2020), 107-153.

- F. Cantero, M. Silvero, *Almost-extreme Khovanov spectra*, Selecta Math. 27, Article 95 (2021).

- M. I. de Frutos-Fernández, D. Macías Castillo, D. Martínez Marqués, *The refined class number formula for Drinfeld modules*, preprint available at second author's webpage.

- R. Lipshitz and S. Sarkar, *A Khovanov stable homotopy type*, JAMS, 27 (4) 983-1042 (2014)

- L. Piccirilo, *The Conway knot is not slice*, Annals of Math. 191 (2) 581-591 (2020)

- B. Mazur, J. Tate, J. Teitelbaum, *On p-adic analogues of the conjectures of Birch and Swinnerton-Dyer*, Invent. Math. 84 (1987) 1-48.

- L. Taelman, *Special L-values of Drinfeld modules*, Annals of Math. 175 (2012) 369-391.

Training activities

Our research group has ample experience supervising PhD students. D. Macías is currently supervising two students and F. Cantero is supervising one. We encourage interaction between all students of members of the group. We organise several research seminars and informal study groups, including the following. Most weeks, several of them have a session.

-ICMAT-UAM Number Theory Seminar: D. Macías is an organiser of this seminar, which regularly hosts international experts on topics of Arithmetic Geometry and Number Theory.

-Study group on Iwasawa theory: Organised by D. Macías, this study group allows students to explain topics within Arithmetic Geometry that they have been studying.

-Study group on Drinfeld and Anderson modules: Organised by D. Macías, this advanced study group discusses modern research articles about Arithmetic Geometry in positive characteristic.

-Study group in Algebraic Topology: Organised by F. Cantero, aimed at PhD students.

-ICMAT Geometry seminar: Organised by Benjamin Bode, Mario García, Óscar García-Prada, Ángel González and Daniel Peralta.

- Madrid Group Theory Seminar: Organised by Leo Margolis and Alejandra Garrido.

-Mathematical Formalization workshop: Organised by M. I. de Frutos, this workshop focuses on using the interactive theorem prover Lean.

-Study group in non-Archimedean and hybrid geometry: Organised by J. I. Burgos.

Training plan

The student should spend the first year acquiring general knowledge within their chosen area, through reading books and eventually research articles. They will discuss it regularly with their advisor and occasionally in the setting of the study groups. For example, for a PhD project in Arithmetic Geometry, and depending on the student's background, this general knowledge could include homological algebra and K-theory and the basic theory of global and local fields, as well as the arithmetic of elliptic curves, abelian varieties or Drinfeld modules. For a PhD project in Algebraic Topology, it could also include homological algebra, as well as knot theory and topological spectra. The above seminars would also be relevant.

In the second year, they will concentrate more tightly on a first PhD problem suggested by their advisor. Eventually, they will be guided in the process of writing a first research article, either as sole authors or in collaboration with their advisor (and, potentially, additional co-authors).

In the remaining years of their PhD, once a first problem has been fully understood and solved, their advisor will help them find natural directions in which to expand their research, to eventually arrive at a successful thesis combined with publications in prestigious journals.

Research stays, visits, conferences and seminars

Both D. Macías and F. Cantero have extensive networks of international collaborators. We will encourage and facilitate research stays at other institutions, so that the student can discuss their work with other experts in their area. For example, for a PhD project in Arithmetic Geometry, we envision possible stays in one or two of the following cities (in no particular order): London, Seoul, Munich, Barcelona, San Diego, Caen or Genova. A project in Algebraic Topology would involve possible stays in Western Ontario, Oregon, Barcelona or Berlin.

We will also encourage and facilitate the student's participation in appropriate international conferences and seminars, both as attendants and, once original research results have been obtained, as speakers.

In order to cover the expenses associated to the above purposes, the student will have access to the funding awarded to the Research Project, as well as to the funds associated to the predoctoral contract.