Logic and Reasoning Research Group
IIIA - CSIC

The Logic and Reasoning research group at the Artificial Intelligence Research Institute (IIIA - CSIC) has a vast experience in the investigation of algebraic aspects of nonclassical logics, with special attention to many-valued and fuzzy ones. In addition, the group has a solid research record concerning their application to uncertain and inconsistent epistemic situations. In this field, the high reputation of the host institution is highlighted by the number and relevance of the scientific publications, and the intense collaboration with several international and renowned research groups.

The IIIA research team on Logic and Reasoning currently includes the following members: Jordi Levy (Head of the department), Francesc Esteva (Professor Ad-Honorem), Tommaso Flaminio, Christian Blum, Lluís Godo, Gonzalo Escalada-Imaz, Pedro Meseguer and Felip Manyà (permanent staff), Pilar Dellunde (Vinculated Staff), Sara Ugolini (Ramón y Cajal), Amanda Vidal (Marie Curie fellow). In the last years, also Félix Bou (postdoctoral researcher), Diego Valota (INdAM-COFUND fellow), Pietro Codara (INdAM-COFUND fellow) and Jesús Giráldez-Cru have been members of the team.

The IIIA - CSIC research group is the leading group of the MSCA-RISE project MOSAIC (https://cordis.europa.eu/project/id/101007627). MOSAIC - Modalities in Substructural Logics: Theory, Methods and Applications is coordinated by Tommaso Flaminio (PI) and it links Spain and other European countries with more than 20 centers of excellence in Argentina, Australia, Brazil, South Africa, and the United States. In addition, it is worth mentioning that the IIIA-CSIC Research Team has created and maintained international scientific collaborations with several research teams, like the ones of Profs. Petr Cintula (Prague, Czech Republic), Carles Noguera and Paolo Aglianò (University of Siena, Italy) Profs. Baaz and Fermüeller (Vienna, Austria), Profs. Di Nola and Spada (Salerno, Italy), Profs. Dubois and Prade (Toulouse, France), Profs. Moraschini, Font, Gispert, Torrens and Verdú (Barcelona, Spain), Prof M. Busaniche, late Prof. R. Cignoli and Prof. R. Rodriguez (Santa Fe and Buenos Aires, Argentina) and Prof. Coniglio (Campinas, Brazil). The bibliographic references in this document witnesses the fruitful collaborations of the IIIA-Research Team with the above-mentioned groups.

The IIIA - CSIC research group, together with other research groups in Barcelona recently formed the Barcino research group (Barcelona research group in nonclassical logics) (https://barcinologic.github.io/web/#). The research of the group is centered on the study of non-classical logics with special attention to their algebraic aspects. The group mainly investigates fuzzy, modal, and intuitionistic logics as well as formal logics for artificial intelligence. The Barcino group is undoubtedly one of the major hubs for research in nonclassical logics in the European Union, and many of its members belong to the IIIA-CSIC group.

Research areas of the IIIA - CSIC

Mathematical Fuzzy Logic

Classical logic formalizes correct reasoning in a framework where predicates can be evaluated to be either true or false. The idea of extending the expressive power of logical deduction to include evaluations beyond the two values started being explored within formal systems in the 1920s. In particular, one of the first systems of non- classical logics was introduced by Łukasiewicz: initially a three-valued logic, later expanded by himself to a finitely-valued, and then ultimately to an infinite-valued system together with Tarski. Gödel as well introduced a class of finitely-valued systems lying between classical and intuitionistic logics. All these systems can now be studied uniformly in a wider framework of many-valued logics: Mathematical Fuzzy Logic, or MFL.

MFL is a framework of mathematical logic first introduced by the eminent Czech logician P. Hájek in his celebrated 1998 book "Metamathematics of Fuzzy Logic". MFL is meant to formalize predicates whose truth values range in the real unit interval [0,1], and whose semantics is based on the choice of a conjunction connective represented by a binary operation called t-norm. Hájek's logics are often referred to as systems of "truth-degrees", and are meant to formalize predicates that cannot be reduced to be either fully true nor false (e.g., "today is cold", "I am tall"). While Hájek bases his systems on continuous t-norms, later on Esteva and Godo, from the IIIA - CSIC, prove that it is sufficient and necessary for the t-norm to be left-continuous in order to have a standard interpretation of every other connective of the logics. They define the logic MTL, later proved by Jenei an Montagna to be indeed the logic of left-continuous t-norms, which is a wide framework to study mathematical fuzzy logics uniformly. These logics are algebraizable in the sense of Blok and Pigozzi, thus can be studied with algebraic techniques, and therefore have been deeply investigated by the community of algebraic logicians. Since Hájek's book, the community working on MFL has grown to include researchers working in various disciplines, such as algebraic logic, philosophical foundations, proof theory, complexity, and even AI knowledge representation techniques. Mathematical fuzzy logics are also interesting examples of substructural logics, a wider framework that we shall discuss later on in more detail.

The IIIA - CSIC group has significantly contributed to the current understanding of MFL, introducing for instance novel approaches to study large classes of fuzzy logics based on algebraic constructions and geometrical methods.

Substructural logics

Non-classical logics are formal frameworks which started to arise in the 20th century in order to interpret different aspects of reasoning and of the notion of truth, with respect to which classical logic is inadequate. Among the most well-known we find, e.g., intuitionistic logic, fuzzy logics, relevance logics, linear logic. All of the mentioned systems can be seen uniformly in the wide framework of substructural logics, a class of systems definable as axiomatic extensions of the Full Lambek Calculus, a formal system generating from the groundbreaking ideas by Lambek who formalizes the syntax of natural language in a calculus. The framework of substructural logics is therefore extremely strong for its comparative potential. Moreover, substructural logics are all *algebraizable*, which essentially means that logical deducibility is fully and faithfully interpreted in the algebraic consequence relation. Therefore, logical properties can be studied algebraically and vice

versa. From an algebraic point of view, their corresponding structures (residuated lattices) have amenable algebraic properties that make them especially tractable in the setting of Universal Algebra, made powerful by the celebrated work of Birkhoff. As a result, interesting *bridge theorems* connecting logical and algebraic properties are obtained in this general framework. Moreover, deep connections with other relevant mathematical theories are also often extremely useful and thus used in this context. The literature is rich with results that connect our structures of interest to topological spaces, geometrical objects, group theory, functional analysis, via categorical equivalences and dualities. Thanks to all the fruitful techniques and connections, most researchers working in algebraic logic nowadays work in this framework.

Researchers at the IIIA - CSIC have introduced and studied novel algebraic constructions and methods to study large classes of residuated lattices from a uniform perspective.

Foundations of the probability theory of many-valued events

Probability theory, as founded by the work of Kolmogorov, gives a suitable framework to formalize uncertainty, dealing with the likelihood of the occurrence of events. Classical probability theory only addresses events whose truth or falseness can, at some moment, be fully established. However, the intrinsic vagueness in many real-life declarative statements, such as “There is traffic”, or “It is going to be cold”, requires formal systems in which partial truth can be handled. We refer to events that may turn out to be partially true as many-valued events, in opposition to classical events which can only be either true or false. MFL is a suitable logical framework to deal with partial truths, and has indeed shown to be a successful framework for the foundations of the probability theory of many-valued events. In Łukasiewicz logic in particular, a notion of probability map has been given by a celebrated work by Mundici with state maps of 1995, which are in one-one correspondence to Borel probability measures via the so-called Kroupa-Panti representation theorem, which views state maps as Lebesgue integrals over Łukasiewicz formulas. Here, algebraic finite additivity corresponds to measure theoretic countable additivity, making the algebraic framework suitable and powerful to study probability theory.

Recent developments in the geometric description of uncertainty have been obtained by the IIIA - CSIC group and will be recently published in a couple of papers that will appear in the Annals of Pure and Applied Logic.

Selected Publications of the IIIA - CSIC Team

(last 5 years)

1. P. Aglianò, S. Ugolini (2023). Projectivity and unification in substructural logics of generalized rotations, International Journal of Approximate Reasoning 153, 172-192.

2. G. Rosella, T. Flaminio, S. Bonzio. Counterfactuals as modal conditionals, and their probability. Artificial Intelligence 323: 103970, 2023.

3. E. A. Corsi, T. Flaminio, H. Hosni. A logico-geometric comparison of coherence for non- additive uncertainty measures. *Annals of Pure and Applied Logic* 103342, https://doi.org/ 10.1016/j.apal.2023.103342. 2023

4. T. Flaminio, S. Ugolini. Encoding de Finetti's coherence within Łukasiewicz logic and MV- algebras. *Annals of Pure and Applied Logic* 103337, https://doi.org/ 10.1016/ j.apal.2023.103337. 2023

5. P. Aglianò, S. Ugolini (2023). Projectivity in (bounded) commutative integral residuated lattices, Algebra Universalis 84(2), 2023.

6. Juan C. L. Teze, Lluís Godo, & Gerardo I. Simari (2022). An Approach to Improve Argumentation-Based Epistemic Planning with Contextual Preferences. International Journal of Approximate Reasoning, 151, 130-163.

7. Tommaso Flaminio, Lluís Godo, & Sara Ugolini (2022). An Approach to Inconsistency-Tolerant Reasoning About Probability Based on Łukasiewicz Logic. F. Dupin al. (Eds.), SUM 2022 (pp. 124–-138). Springer.

8. Chu-Min Li, Zhenxing Xu, Jordi Coll, Felip Manyà, Djamal Habet, & Kun He (2022). Boosting branch-and-bound MaxSAT solvers with clause learning. AI Communications, 35, 131-151

9. Tommaso Flaminio, Angelo Gilio, Lluís Godo, & Giuseppe Sanfilippo (2022). Canonical Extensions of Conditional Probabilities and Compound Conditionals. Davide Ciucci al. (Eds.), 17th Intl. Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU 2022) (pp. 584--597). Springer

10. Tommaso Flaminio, Angelo Gilio, Lluís Godo, & Giuseppe Sanfilippo (2022). Compound Conditionals as Random Quantities and Boolean Algebras. Proceedings of the 19th International Conference on Principles of Knowledge Representation and Reasoning, KR2022, Haifa, Israel. July 31 - August 5, 2022 (pp. 141-151).

11. Marco Cerami, & Francesc Esteva (2022). On decidability of concept satisfiability in Description Logic with product semantics. Fuzzy Sets and Systems, 445, 1-21.

12. Marcelo Coniglio, Francesc Esteva, Tommaso Flaminio, & Lluís Godo (2022). On the expressive power of Lukasiewicz's square operator. Journal of Logic and Computation, 32, 767-807.

13. Tommaso Flaminio, & Umberto Rivieccio (2022). Prelinearity in (quasi-)Nelson logic. Fuzzy Sets and Systems, 445, 66-89.

14. Francesc Esteva, Lluís Godo, & Sandra Sandri (2021). A similarity-based three-valued modal logic approach to reason with prototypes and counterexamples. MJ. Lesot, & C. Marsala (Eds.), Fuzzy Approaches for Soft Computing and Approximate Reasoning: Theories and Applications (pp 45-59). Springer.

15. Tommaso Flaminio, Lluís Godo, & Sara Ugolini (2021). Canonical Extension of Possibility Measures to Boolean Algebras of Conditionals. 16th European Conference, ECSQARU 2021, Prague, Czech Republic, September 21-24, 2021, Proceedings (pp. 543--556). Springer

16. Marcelo Coniglio, Francesc Esteva, Joan Gispert, & Lluís Godo (2021). Degree-preserving Gödel logics with an involution: intermediate logics and (ideal) paraconsistency. O. Arielli, & A. Zamansky (Eds.), Arnon Avron on Semantics and Proof Theory of Non-Classical Logics (pp 107--139). Springer.

17. Francesc Esteva, Aldo Figallo-Orellano, Tommaso Flaminio, & Lluís Godo (2021). Logics of formal inconsistency based on distributive involutive residuated lattices. Journal of Logic and Computation, 31, 1226-1265.

18. Tommaso Flaminio (2021). On standard completeness and finite model property for a probabilistic logic on Łukasiewicz events. Int. J. Approx. Reason., 131, 136–150

19. Amanda Vidal (2021). On transitive modal many-valued logics. Fuzzy Sets and Systems, 407, 97-114.

20. Paolo Aglianò, & Sara Ugolini (2021). Strictly join irreducible varieties of residuated lattices. Journal of Logic and Computation, 32, 32-64

21. Amanda Vidal, Francesc Esteva, & Lluís Godo (2020). Axiomatizing logics of fuzzy preferences using graded modalities. Fuzzy Sets and Systems, 401, 163-188.

22. Tommaso Flaminio, Lluís Godo, & Hykel Hosni (2020). Boolean algebras of conditionals, probability and logic. Artificial Intelligence, 286, 103347

23. Tommaso Flaminio, & Sara Ugolini (2020). Hyperstates of Involutive MTL-Algebras that Satisfy $(2x)^2=2(x^2)$. Shier Ju, Alessandra Palmigiano, & Minghui Ma (Eds.), Nonclassical Logics and Their Applications (pp. 1--14). Springer Singapore

24. Tommaso Flaminio (2020). Three Characterizations of Strict Coherence on Infinite-Valued Events. The Review of Symbolic Logic, 1–18.

25. Tommaso Flaminio, Lluís Godo, & Ricardo Oscar Rodriguez (2019). A Representation Theorem for Finite Godel Algebras with Operators. 11th conference of the European Society for Fuzzy Logic and Technology, EUSFLAT-2019

26. Tommaso Flaminio, Lluís Godo, & Ricardo Oscar Rodriguez (2019). A Representation Theorem for Finite Gödel Algebras with Operators. 26th Workshop on Logic, Language, Information and Computation, WoLLIC 2019 (pp. 223-235). Springer.

27. Francesc Esteva, Tommaso Flaminio, & Lluís Godo (2019). From Fuzzy Sets to Mathematical Fuzzy Logic. Archives for Soft Computing, 2, 26-59

28. Marcelo Coniglio, Francesc Esteva, Joan Gispert, & Lluís Godo (2019). Maximality in finite-valued Lukasiewicz logics defined by order filters. Journal of Logic and Computation, 29, 125-156.

29. Amanda Vidal, Francesc Esteva, & Lluís Godo (2018). An alternative axiomatization for a fuzzy modal logic of preferences. ESTYLF 2018 (pp. 370-378). Universidad de Granada.

30. Tommaso Flaminio, Lluís Godo, & Sara Ugolini (2018). Towards a probability theory for product logic: states, integral representation and reasoning. International Journal

of Approximate Reasoning, 93, 199-218

31. Pere Pardo, & Lluís Godo (2018). A temporal argumentation approach to cooperative planning using dialogues. Journal of Logic and Computation, 28, 551-580.

32. Pietro Codara, Francesc Esteva, Lluís Godo, & Diego Valota (2018). Connecting systems of mathematical fuzzy logic with fuzzy concept lattices. J. Medina al. (Eds.), Information Processing and Management of Uncertainty in Knowledge-Based Systems. Theory and Foundations. IPMU 2018.

33. T. Flaminio, H. Hosni, F. Montagna, Strict Coherence on Many-Valued Events. The Journal of Symbolic Logic 83(1): 55–69, 2018.

3. RESEARCH PROJECTS (a selection of the last 5 years):

1. SHORE - The Shape of Reasoning: Many-valued Logics and Uncertainty. Spanish Ministry of Science and Innovation (PID2022-141529NB-C22). **PI**: Tommaso Flaminio

2. ASUBS 2023-2026 Algebraic constructions in substructural logics. Grant “Ayudas attracción de talento Ramón y Cajal 2021”, cofunded by CSIC and IIIA. Spanish National project (20235AT019). **PI**: Sara Ugolini (Barcelona, Spain).

3. LAC 2022 Lógicas y Algebras para Condicionales. Proyecto Intramural Especial (OEP 2019). **PI**: Tommaso Flaminio (Barcelona, Spain).

4. JAE-Intro 2022 Algebraic methods for reasoning under uncertainty. **Supervisor**: Tommaso Flaminio (Barcelona, Spain).

5. MOSAIC 2021–2026 Modalities in Substructural Logics: Theory, Methods and Applications. H2020-MSCA-RISE-2020 (Grant Agreement number 101007627). **Int. Coordinator**: Tommaso Flaminio (Barcelona, Spain).

6. SuMoL 2020–2022 Substructural modal logics for Knowledge Representation. CSIC I-Link project (LINKC20018). **PIs**: Tommaso Flaminio (Barcelona, Spain) and Igor Sedlar (Prague, Czech Republic).

7. ISINC 2020 : Inference Systems for Inconsistent Information: logical foundations. Spanish Ministry of Science and Innovation. **PI**: Lluis Godo

8. PROOFS : Practical Proof Systems Beyond Resolution. Proyectos de I+D+i orientada a retos de la sociedad. **PI**: Jordi Levi

9. LOGISTAR : Enhanced data management techniques for real time logistics planning and schedulingHorizon 2020 (Grant Agreement No. 76914). **PI**: Jordi Levi